Pipe Flow/Friction Factor Calculations using Excel Spreadsheets

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Introduction

The Darcy-Weisbach equation or the Fanning equation and the friction factor (Moody friction factor or Fanning friction factor) are used for a variety of pressure pipe flow calculations. Many of these types of calculations require a graphical and/or iterative solution. The necessary iterative calculations can be carried out conveniently through the use of a spreadsheet. This tutorial starts with discussion of the Darcy-Weisbach and the Fanning equations along with the parameters contained in them and the U.S. and S.I. units typically used in the equations. Several example calculations are included and spreadsheet screenshots are presented and discussed to illustrate the ways that spreadsheets can be used for pressure pipe flow/friction factor calculations, including both laminar and turbulent flow and cases in which minor losses are included in the calculations.

1. The Basic Equations

The Darcy-Weisbach equation and the Fanning equation are two flexible, widely used relationships for pressure pipe flow calculations. The Darcy-Weisbach equation, in its most widely used form (including minor losses and with uniform pipe diameter and reference velocity for Ks) is:

$$h_L = \left[f_m\left(\frac{L}{D}\right) + \Sigma K\right]\left(\frac{V^2}{2g}\right)$$

This is a semi-empirical equation, but it is dimensionally consistent, so it has no dimensional constants and any consistent set of units can be used. The parameters in the equation are defined below along with their commonly used U.S. and S.I. units.

L is the pipe length, in ft (U.S.) or m (S.I.)

D is the pipe diameter, in ft (U.S.) or m (S.I.)

V is the average velocity of the fluid flowing through the pipe, in ft/sec (U.S.) or m/s (S.I.). Note that **V** is defined as **V = Q/A**, where **Q** is the volumetric flow rate of the fluid and **A** is the cross-sectional area of flow.

$\mathbf{h_L}$ is the frictional head loss due to fluid flowing at an average velocity, **V**, through a pipe of length, **L**, and diameter, **D**, with Moody friction factor equal to $\mathbf{f_m}$. The frictional head loss will be in ft for U.S. units and in m for S.I. units.

g is the acceleration due to gravity. (**g** = 32.17 ft/sec^2 = 9.81 m/s^2)

$\mathbf{f_m}$ is the Moody friction factor, which is dimensionless and is a function of Reynolds number (Re = DVρ/μ) and relative roughness (**ε/D)**. Note that this parameter is also called the Darcy friction factor. The two terms can be used interchangeably. The m subscript is usually not present on the symbol for the Moody friction factor. It is being used in this tutorial to differentiate it from the Fanning friction factor, which will be introduced shortly.

ε is an empirical pipe roughness parameter, in ft for U.S. or mm for S.I. units.

ΣK is the sum of the minor loss coefficients for the pipe system. Minor loss coefficients are dimensionless. They are used to account for frictional head loss or frictional pressure drop due to pipe fittings, changes in cross-section, entrances and exits. Typical K values for common fittings are available in many handbooks, textbooks and websites. Table 1 below shows some typical values.

Fitting	Loss Coefficient, K
Globe valve, fully open	10.0
Angle valve, fully open	5.0
Swing check valve, fully open	2.5
Gate valve, fully open	0.2
Short-radius elbow	0.9
Medium-radius elbow	0.8
Long-radius elbow	0.6
45 degree elbow	0.4
Close return bend	2.2
Standard tee, flow through run	0.6
Standard tee, flow through branch	1.8
Square entrance	0.5
Exit	1.0

Table 1. Typical Values of Minor Loss Coefficients

For a table with additional minor loss coefficient values, see:

Perry's Chemical Engineer's Handbook, 8th Ed. Table 6-4

For more discussion of the Darcy-Weisbach equation, see:

Standard Handbook for Civil Engineers, 5th Ed, Sec. 21.30. Darcy-Weisbach Formula

A common form of the Fanning equation (including minor losses and with uniform pipe diameter and reference velocity for Ks) is:

$$4\, f_f \left(\frac{L}{D}\right) + \Sigma K = \frac{2\, \Delta P_f}{\rho V^2}$$

Like the Darcy-Weisbach equation, the Fanning equation is also a semi-empirical, dimensionally consistent equation. The parameters D, V, ΣK, and L are the same as in the Darcy-Weisbach equation just described above. The other parameters in the Fanning equation are as follows:

ρ is the density of the flowing fluid in slugs/ft^3 for U.S. or kg/m^3 for S.I. units.

ΔP_f is the frictional pressure drop due to the flowing fluid in lb/ft^2 for U.S. or Pa for S.I. units. (Note that lb is being used for a unit of force and lb_m as a unit of mass in this tutorial.)

f_f is the Fanning friction factor, which is dimensionless and is a function of Reynolds number and **ε/D**. The relationship between the Fanning friction factor and the Moody friction factor is: **f_m** = 4 **f_f**. Note that the m and f subscripts are not typically used. The symbol **f** is commonly used for both the Moody friction factor and the Fanning friction factor. The subscripts are being used in this book to avoid confusion between the two different friction factors, which are both in common use.

For more discussion of the Fanning equation, see: Perry's Chemical Engineers' Handbook, 8th Ed. Sec 6.1.4. Incompressible Flow in Pipes and Channels

Values of **ε** for common pipe materials are available in many handbooks and textbooks, as well as on a variety of websites. Table 2 below shows some typical values.

Pipe Material	Roughness, ε	Roughness, ε
	ft	mm
Drawn brass or copper	0.000005	0.00152
Commercial steel	0.00015	0.0457
Wrought iron	0.00015	0.0457
Asphalted Cast Iron	0.0004	0.122
Galvanized Iron	0.0005	0.152
Wood Stave	0.002	0.183 - 0.914
Concrete	0.004	0.305 - 3.05
Riveted Steel	0.006	0.914 - 9.14

Table 2. Values of Pipe Roughness, **ε**

The surface roughness values in Table 1 came from the following two sources:

Perry's Chemical Engineers' Handbook, 8th Ed., Table 6-1 - S.I. units

Mark's Standard Handbook for Mechanical Engineers, 11th Ed., Table 3.3.9 - U.S. units

You may have noticed that the Fanning equation is written in terms of frictional pressure drop, while the Darcy-Weisbach equation is written in terms of frictional head loss. The relationship between these two measures of frictional loss is as follows:

$$\Delta P_f = \rho g h_L = \gamma h_L$$

The parameters in this equation are as follow:

h_L is the frictional head loss in ft (U.S.) or m (S.I.) as defined above

ΔP_f is the frictional pressure drop in lb/ft^2 (U.S.) or Pa (S.I.).

ρ is the density of the flowing fluid in slugs/ft^3 (U.S.) or kg/m^3 (S.I.)

γ is the specific weight of the flowing fluid in lb/ft^3 (U.S.) or N/m^3 (S.I.)

g is the acceleration due to gravity = 32.17 ft/sec^2 = 9.81 m/s^2.

The Darcy-Weisbach equation and the Fanning equation can both be used only for fully developed, pressure flow (either laminar or turbulent) in a pipe, piping system, or closed conduit with a non-circular cross-section. The next two chapters contain discussion of laminar and turbulent flow and of the meaning of fully developed flow.

2. Laminar and Turbulent Flow in Pipes

Determination of whether a given flow is laminar or turbulent is important for several types of fluid flow situations. Here we will be concerned in particular with pressure flow in pipes and whether a given flow is laminar or turbulent.

In general, laminar flow is characterized by a lack of turbulence to cause mixing within the fluid and will be present with low fluid velocity and/or high fluid viscosity. Turbulent flow, however, has turbulence and mixing within the flow and takes place with high fluid velocity and/or low fluid viscosity. Differences between laminar and turbulent flow are illustrated in the diagrams below.

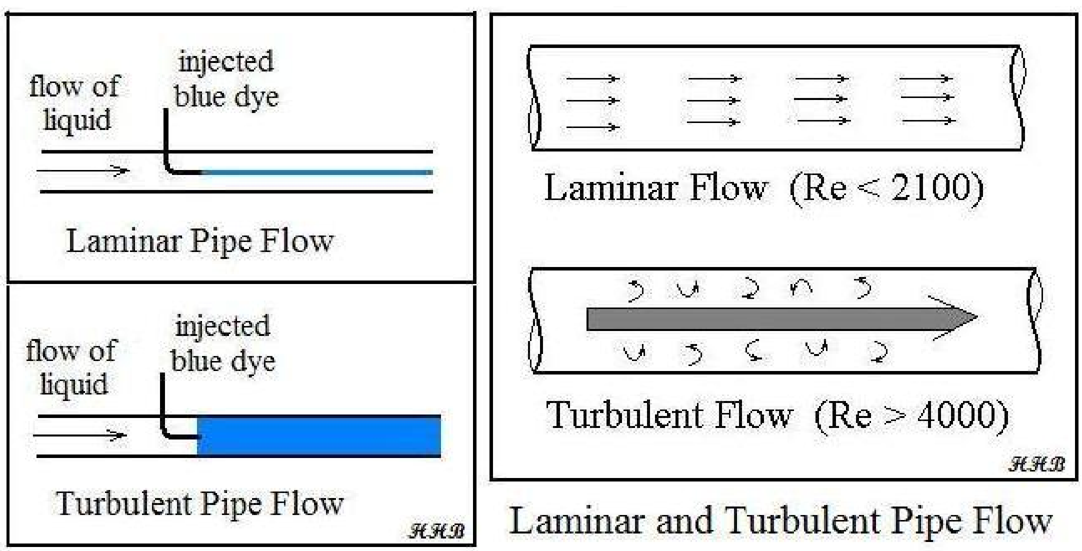

Figure 2. Laminar and Turbulent Pipe Flow

Osborne Reynolds, a pioneer in the study of differences between laminar and turbulent flow, performed his experiments in the late 1800s. He injected dye into fluids flowing through pipes under varied conditions and came up with a group of variables to predict whether pipe flow would be laminar or turbulent. The group of variables, $DV\rho/\mu$ (more details later) came to be known as the Reynolds number.

Reynolds' experiments are illustrated by the diagram at the left above, showing that under laminar flow conditions the dye flows in streamlines and doesn't mix with the rest of the fluid in the pipe, as shown in the upper part of the diagram. The lower diagram illustrates turbulent flow, in which fluid turbulence mixes the dye throughout the flowing fluid. The diagrams at the right are schematic illustrations of laminar flow with straight streamlines and no turbulence and turbulent flow with eddy currents that mix the flowing fluid.

The Reynolds number mentioned above is defined for pressure flow in pipes as follows:

$$\mathbf{Re = DV\rho/\mu}$$

The parameters in the equation and typical U.S. and S.I. units are:

D – the diameter of the pipe (ft – U.S. or m – S.I.)

V – the average velocity of the fluid flowing in the pipe (ft/sec – U.S. or m/s – S.I.) (The average velocity is defined as **V = Q/A**, where **Q** is the volumetric flow rate through the pipe and **A** is the cross-sectional area of flow.)

ρ – the density of the flowing fluid (slugs/ft^3 – U.S. or kg/m^3 – S.I.)

μ – the dynamic viscosity of the flowing fluid (lb-sec/ft^2 – U.S. or N-s/m^2 – S.I.)

The generally accepted criteria currently in use for laminar and turbulent flow in pipes are as follow:

For **Re < 2300** the flow will be **laminar**

For **Re > 4000** the flow will be **turbulent**

For **2300 < Re < 4000** the flow **may be either laminar or turbulent**, depending upon other factors such as the roughness of the pipe surface and the pipe entrance conditions. This is called the transition region.

Pipe flow for transport of water, air or similar fluids is typically turbulent. Flow of highly viscous fluids, such as lubricating oils, is often laminar. Since density and viscosity are parameters in the Reynolds number, values for **ρ** and **μ** for the flowing fluid are needed for pipe flow calculations. Values of density and viscosity for water from 32^{o}F to 70^{o}F are given in the table below for use in example calculations in this tutorial.

To obtain density and viscosity values for a wide range of liquids see:

Table 2-32 in Perry's Chemical Engineers' Handbook, 8th Ed., for density values, and

Table 2-313 in Perry's Chemical Engineers' Handbook, 8th Ed., for viscosity values

Click below for a spreadsheet from which values of density and viscosity can be obtained for any from a list of 73 liquids.

AccessEngineering Excel Spreadsheet for Incompressible Flow in Pipes and Channels

Temperature, °F	Density, slugs/ft^3	Dynamic Viscosity, lb-s/ft^2
32	1.940	3.732×10^{-5}
40	1.940	3.228×10^{-5}
50	1.940	2.730×10^{-5}
60	1.938	2.334×10^{-5}
70	1.936	2.037×10^{-5}

Table 3. Density and Viscosity of Water

Example #1: Determine whether the flow will be laminar or turbulent for flow of water at 60°F at 0.8 cfs through a 6 inch diameter pipe.

Solution: For water at 60°F, the density and viscosity are: $\boldsymbol{\rho}$ = 1.938 slugs/ft^3 and $\boldsymbol{\mu}$ = 2.334×10^{-5} lb-sec/ft^2.

The average velocity will be: $\mathbf{V = Q/A = Q/(\pi D^2/4)} = 0.8/(\pi*(6/12)^2/4) = 4.07$ ft/sec

The Reynolds number can now be calculated:

$\mathbf{Re = DV\rho/\mu}$ = ((6/12) ft)(4.07 ft/sec)(1.938 slugs/ft^2)/(2.334×10^{-5} lb-sec/ft^2)

$\mathbf{Re = 1.69 \times 10^6}$

Thus the flow is turbulent.

Note that the units all cancel out giving a dimensionless number when you use the fact that 1 lb = 1 slug-ft/sec^2.

3. Fully Developed Flow and the Entrance Region

The entrance region for pipe flow is the portion of the pipe near the entry end, in which the velocity profile is changing. This is illustrated in the diagram below for fluid entering the pipe with a uniform velocity profile. Near the entrance, the fluid in the center of the pipe remains uniform and is not affected by friction between the fluid and the pipe wall. In the fully developed flow region (past the entrance region) the velocity profile has reached its final shape and no longer changes.

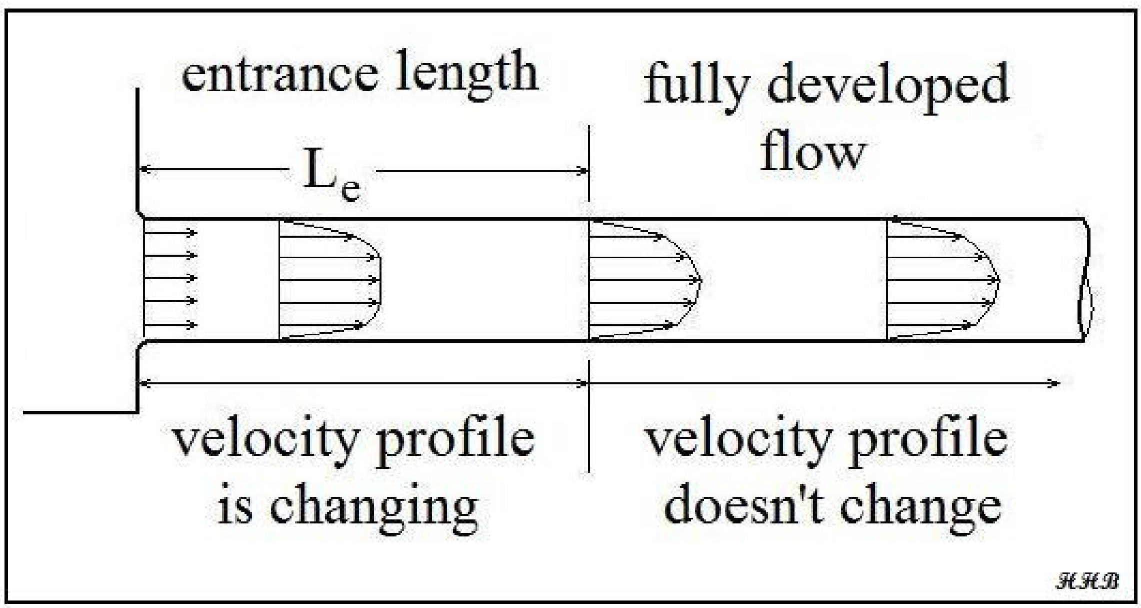

Figure 3. The Entrance Length and Fully Developed Flow

As noted above, the Darcy Weisbach equation and the Fanning equation apply only to fully developed flow, which is the region past the entrance length. If the entrance length is short in comparison with the total length of the pipe for which calculations are being made, then the effect of the entrance region is commonly neglected and the total pipe length is used for calculations. Additional head loss or pressure drop due to the entrance region is typically calculated using an entrance loss coefficient.

The entrance length, L_e, can be estimated for known Reynolds number as follows:

For laminar flow ($Re < 2300$): $\mathbf{L_e/D = 0.06\ Re}$

For turbulent flow ($Re > 4000$): $\mathbf{L_e/D = 4.4\ Re^{1/6}}$

Example #2: Estimate the entrance length for water at 60°F flowing at a rate of 0.8 cfs through a 6 inch diameter pipe (the flow conditions of Example #1).

Solution: As calculated in Example #1, $Re = 1.69 \times 10^6$. This flow is turbulent since Re > 4000, so:

$$\mathbf{L_e/D = L_e/0.5} = 4.4(1.69 \times 10^6)^{1/6} = 48.02$$

Solving for Le gives:

$$\mathbf{L_e} = (0.5)(48.02) = \underline{\mathbf{24.0\ ft}}$$

4. Obtaining Friction Factor Values

A value of the Moody friction factor is needed for most calculations with the Darcy Weisbach equation and a value of the Fanning friction factor is needed for most calculations with the Fanning equation. The exception is when the friction factor is being determined empirically by measuring all of the other variables in the equation.

The use of a Moody friction factor diagram (Figure 4 below) or a Fanning friction factor diagram (Figure 5 below), has been a standard method of obtaining values for f, since the Moody friction factor diagram was first presented by L.F. Moody in his 1944 paper in the *Transactions of the ASME* (Ref #1). As shown in the diagrams, f is a function of Re and/or ε/D. These friction factor diagrams are widely available in books and on websites.

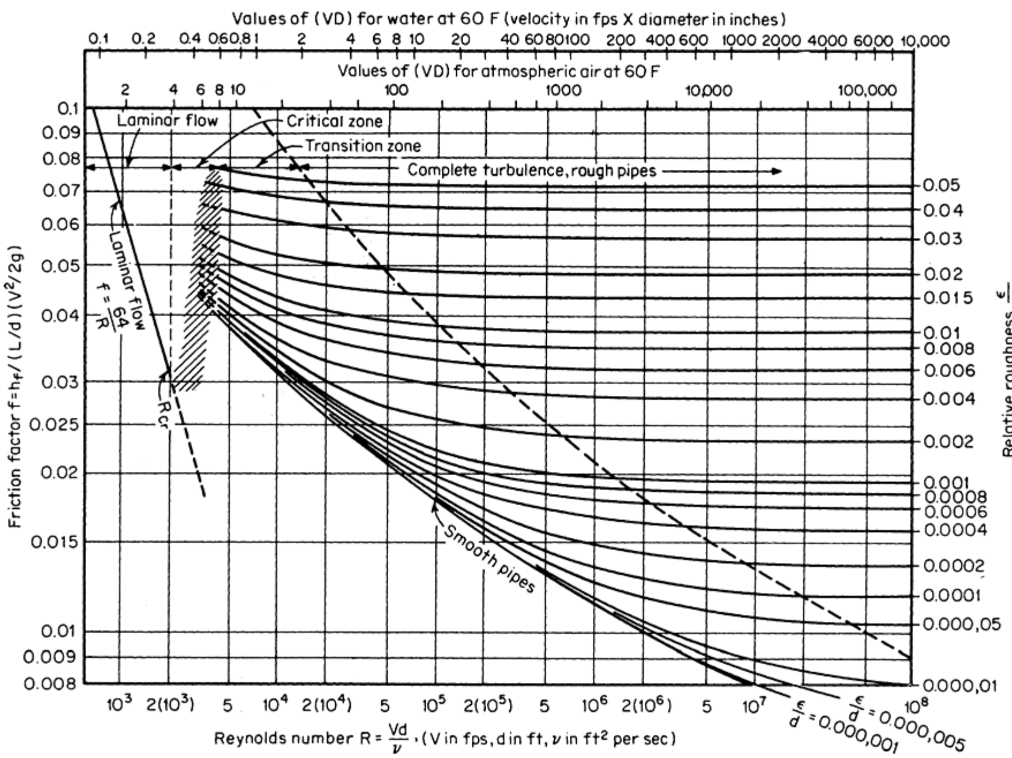

Figure 4. Moody Friction Factor Diagram

Source: [Mark's Standard Handbook for Mechanical Engineers, 11th Ed, Fig 3.3.24]

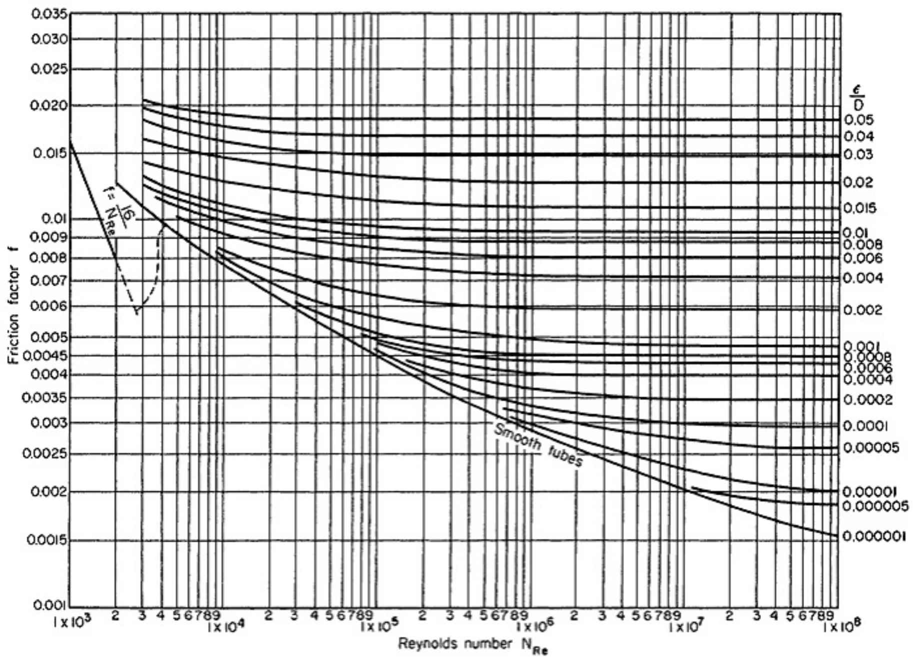

Figure 5. Fanning Friction Factor Diagram

Source: [Perry's Chemical Engineers' Handbook, 8th Ed. Fig. 6-9]

Note that there are four regions identified in the Moody Diagram of Figure 4 as follows:

a) **Laminar Flow**: This is the straight line for **Re** < 2300 at the left side of the Moody Diagram.

b) **Smooth Pipe Turbulent Flow**: This is the dark curve labeled "smooth pipe" in the Moody diagram. Note that f is a function of **Re** only for smooth pipe turbulent flow.

c) **Completely Turbulent Flow**: This is the region of the diagram above and to the right of the dashed line labeled "complete turbulence." Note that **f** is a function of **ε/D** only in this region.

d) **Transition Flow**: This is the part of the diagram between the "smooth pipe" curve and the "complete turbulence" dashed line. Note that **f** is a function of both **Re** and **ε/D** for transition flow.

Example #3: a) Use the Moody friction factor diagram (Fig. 4) to find the value of the Moody friction factor for flow in a pipe with **Re** = 3×10^6 and **ε/D** = 0.01.

b) Use the Fanning friction factor diagram (Fig. 5) to find the value of the Fanning friction factor for the same flow conditions.

Solution: a) From the Moody Diagram in Figure 4, it can be seen that **Re** = 3×10^6 and **ε/D** = 0.01 corresponds to $\mathbf{\underline{f_m = 0.038}}$. Note that this is "completely turbulent flow."

b) From the Fanning Diagram in Figure 5, it can be seen that **Re** = 3×10^6 and **ε/D** = 0.01 corresponds to $\mathbf{\underline{f_f = 0.0095}}$. Note that, as expected, $\mathbf{f_m} = 4\mathbf{f_f}$.

There is an alternative to graphical determination of the friction factor from a Moody diagram or a Fanning diagram. There are equations for the friction factor in terms of **Re** and/or **ε/D.**

Laminar Flow: As shown on the diagrams the friction factor can be calculated quite simply for laminar flow using the equations: $\mathbf{f_m = 64/Re}$ and $\mathbf{f_f = 16/Re}$.

Turbulent Flow: The most general equation for friction factor under turbulent flow conditions is the Colebook equation, shown below in its Moody friction factor form and in its Fanning friction factor form. The turbulent flow portions of the Moody friction factor diagram and the Fanning friction factor diagram are plots of the Colebrook equation.

$$\mathbf{f_m = \{(-2)\log[(\varepsilon/3.7D) + (2.51/(Re^*f_m{}^{1/2})]\}^{-2}} \qquad \text{(Moody friction factor)}$$

$$\text{or: } \mathbf{f_f = \{(-4)\log[(\varepsilon/3.7D) + (1.256/(Re^*f_f{}^{1/2})]\}^{-2}} \qquad \text{(Fanning friction factor)}$$

A difficult aspect of the Colebrook equation in either of the forms shown above is that it cannot be solved explicitly for the friction factor ($\mathbf{f_m}$ or $\mathbf{f_f}$), so an iterative solution is required to calculate the friction factor for known values of **Re** and **ε/D**.

One of several approximations to the Colebrook equation is the Churchill equation, shown below in its Moody and Fanning friction factor forms. These equations do not agree quite as well with the friction factor diagrams over the entire turbulent flow range, but have the advantage of being explicit for the friction factor ($\mathbf{f_m}$ or $\mathbf{f_f}$).

$$\mathbf{f_m = \{(-2)\log[(0.27\varepsilon/D) + (7/Re)^{0.9}]\}^{-2}} \qquad \text{(Moody friction factor)}$$

$$\text{or: } \mathbf{f_f = \{(-4)\log[(0.27\varepsilon/D) + (7/Re)^{0.9}]\}^{-2}} \qquad \text{(Fanning friction factor)}$$

This presents the possibility of using the Churchill equation to obtain an initial estimate of the friction factor and then use that estimate as the starting point for an iterative solution with the Colebrook equation. This process works quite well and converges rapidly to a solution as illustrated in the next two examples.

For further discussion of the Colebrook equation and the Churchill equation see:

Perry's Chemical Engineers' Handbook, 8th Ed. Equations (6-38) and (6-39) and

Mark's Standard Handbook for Mechanical Engineers, 11th Ed, Sec 3.3.11. Flow in Pipes

Example #4: Calculate the value of the Moody friction factor for flow in a pipe with **Re** = 1.2×10^5 and **ε/D** = 0.0075, using the Churchill and Colebrook equations.

Solution: Starting with the Churchill equation, an initial estimate of $\mathbf{f_m}$ is:

$$\mathbf{f_m = \{(-2)log[(0.27*0.0075) + (7/120{,}000)^{0.9}]\}^{-2} = 0.02100}$$

Now, substituting that value of $\mathbf{f_m}$ into the Colebrook equation along with the given values of **ε/D** and **Re** gives:

$$\mathbf{f_m = \{(-2)log[0.0075/3.7) + (2.51/(120{,}000*0.02100^{1/2})]\}^{-2} = 0.020842}$$

Since the value of $\mathbf{f_m}$ calculated with the Colebrook equation is not the same as that calculated with the Churchill equation, the process is repeated again, using the new estimate of $\mathbf{f_m}$:

$$\mathbf{f_m = \{(-2)log[(0.0075/3.7) + (2.51/(3000000*0.02842^{1/2})]\}^{-2} = 0.020850}$$

This iteration returned a much closer value for f_m, but it is still not quite the same. If the calculation is carried out two more times the same value of fm is obtained for two iterations in a row. At that point the process has converged to a solution, showing that: **f_m = 0.020850.** A screenshot of this iterative calculation in an Excel spreadsheet is shown in Figure 6 on the next page.

Example #5: Calculate the value of the Fanning friction factor for flow in a pipe with **Re** = 4×10^5 and **ε/D** = 0.01, using the Churchill and Colebrook equations.

Solution: The solution will be the same as for the Moody friction factor, except for using the Fanning friction factor form of the Churchill and Colebrook equations.

Using the Churchill equation:

$$\mathbf{f_f = \{(-4)log[(0.27*0.01) + (7/400000)^{0.9}]\}^{-2} = 0.009535}$$

First iteration with Colebrook equation:

$$\mathbf{f_f = \{(-4)log[(0.01/3.7) + (2.51/(3000000*0.009535^{1/2})]\}^{-2} = 0.009522}$$

Second iteration with Colebrook equation:

$$\mathbf{f_f = \{(-4)log[(0.01/3.7) + (2.51/(3000000*0.009522^{1/2})]\}^{-2} = 0.009522}$$

The value of the Fanning friction factor is thus: <u>**f_f = 0.009522.**</u>

The Churchill Equation: $\mathbf{f = \{-2\ log[(0.27\varepsilon/D) + (7/Re)^{0.9}]\}^{-2}}$

The Colebrook Equation: $\mathbf{f = \{(-2)log\ [(e/3.7D) + (2.51/(Re*f^{1/2}))]\}^{-2}}$

Churchill Equation	
Initial estimate of f	0.02100

Colebrook Equation	
1st iteration with Colebrook;	0.020842
2nd iteration with Colebrook;	0.020850
3rd iteration with Colebrook;	0.020849
4th iteration with Colebrook;	0.020850
5th iteration with Colebrook;	0.020850
6th iteration with Colebrook;	0.020850
7th iteration with Colebrook;	0.020850
8th iteration with Colebrook;	0.020850
9th iteration with Colebrook;	0.020850
10th iteration with Colebrook;	0.020850
11th iteration with Colebrook;	0.020850
12th iteration with Colebrook;	0.020850

Figure 6. Iterative Calculation of Moody friction factor for Example #4

An Excel workbook that automates the calculations shown in **Example #4** and **Example #5** may be downloaded from the following link:

AccessEngineering Excel Spreadsheet for Incompressible Flow in Pipes and Channels

The use of this spreadsheet workbook for typical pipe flow calculations is discussed and illustrated with examples in the next three sections.

5. Calculation of Frictional Head Loss and/or Frictional Pressure Drop

This is the classic application of the Darcy-Weisbach and Fanning equations. The frictional head loss and/or frictional pressure drop can be calculated for flow of a fluid of known density and viscosity through a pipe of specified length, diameter, and material by the following step-by-step calculations:

1. If necessary, find the density and viscosity of the fluid at a specified temperature.

2. Calculate the average velocity of the fluid from the specified flow rate and pipe diameter. [$\mathbf{V = Q/A = Q/(\pi D^2/4)}$].

3. Calculate the Reynolds number. [$\mathbf{Re = DV\rho/\mu}$]

4. Determine the value of the surface roughness, **ε**, for the specified pipe material.

5. Calculate the pipe surface roughness ratio, **ε/D**. (Note that **ε** and **D** must be expressed in the same units.)

6. Calculate the Moody friction factor or the Fanning friction factor using the procedure discussed and illustrated in the last chapter using the calculated values of **Re** and ε/D (or just the value of **Re** if the flow is laminar).

7. Calculate the frictional head loss with the Darcy Weisbach equation or the frictional pressure drop with the Fanning equation:

$$h_L = \left[f_m\left(\frac{L}{D}\right) + \Sigma K\right]\left(\frac{V^2}{2g}\right) \qquad 4 f_f\left(\frac{L}{D}\right) + \Sigma K = \frac{2\,\Delta P_f}{\rho V^2}$$

   using the specified or calculated values of $\mathbf{f_f}$ **or** $\mathbf{f_m}$**, L, D, V**, **ΣK** and ρ if necessary,

8. As necessary, calculate the frictional pressure drop or frictional head loss using the relationship: $\mathbf{\Delta P_f = \rho g h_L = \gamma h_L}$**.**

These calculations are illustrated with Example #6.

Example #6: What would be the frictional head loss in ft and the frictional pressure drop in psi for 0.9 cfs of water at 50°F flowing through 80 ft of 8 inch diameter galvanized iron pipe?

Solution: These calculations can be made using the Moody friction factor and the Darcy-Weisbach equation with the 8 steps just listed:

1. At 50°F the density and viscosity of water are: $\rho = 1.94$ slugs/ft^3 and $\mu = 2.72 \times 10^{-5}$ lb-s/ft^2.

2. The average velocity of the water is:

$$\mathbf{V = Q/(\pi D^2/4)} = (0.90)/[\pi(8/12)^2/4] = 2.58 \text{ ft/sec}$$

3. The Reynolds number is:

$$\mathbf{Re = DV\rho/\mu} = (8/12)(2.6)(1.94)/(2.72 \times 10^{-5}) = 120{,}000$$

4. From Table 1, the surface roughness for galvanized iron pipe is $\boldsymbol{\varepsilon} = 0.0005$ ft.

5. The pipe roughness ratio is: $\boldsymbol{\varepsilon/D} = 0.0005/(8/12) = 0.00075$

6. As calculated in Example #4: $\mathbf{f_m} = 0.02085$.

7. Substituting specified and calculated values for the parameters in the Darcy Weisbach equation (with $\boldsymbol{\Sigma K} = 0$) gives:

$$\mathbf{h_L = f(L/D)(V^2/2g)} = (0.02085)(80/0.6667)[2.58^2/(2*32.2)] = \mathbf{0.2585 \text{ ft} = h_L}$$

8. The frictional pressure drop can now be calculated from:

$$\mathbf{\Delta P_f = \rho g h_L} = 1.94*32.174*0.259 = 16.1 \text{ lb/ft}^2$$

$$= 16.1/144 \text{ psi} = \mathbf{0.112 \text{ psi} = \Delta P_f}$$

The spreadsheet screenshot in Figure 7 below shows how this solution can be implemented in a spreadsheet. The input data is entered into the yellow cells and the spreadsheet then calculates $\mathbf{h_L}$ and $\mathbf{\Delta P_f}$. The iterative calculation of $\mathbf{f_m}$ isn't shown on this screenshot.

The calculations using the Fanning friction factor would follow the same pattern, simply using the Fanning friction factor forms for the Churchill equation and the Colebrook equation.

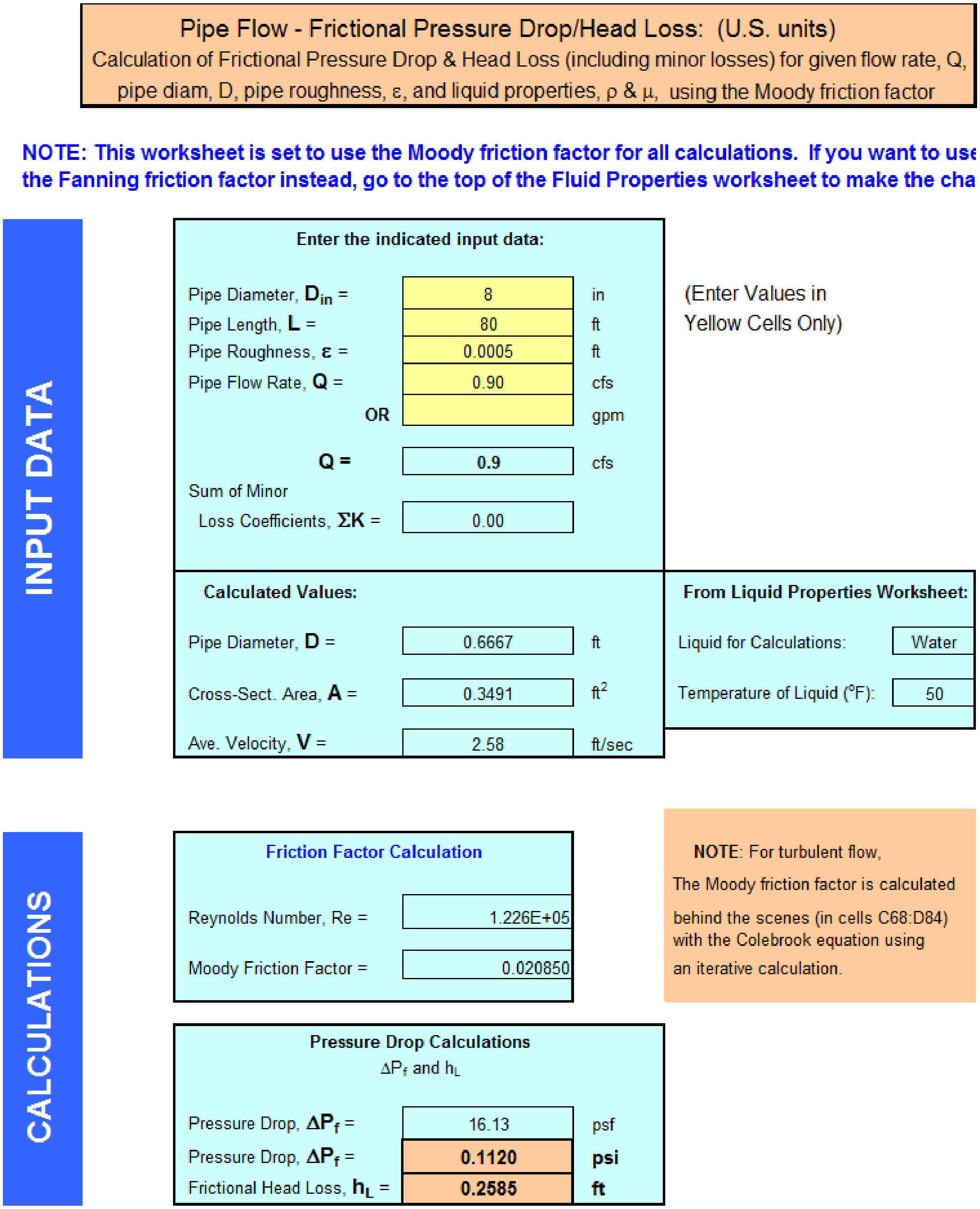

Figure 7. Screenshot of a Frictional Head Loss Calculator Spreadsheet

Source of spreadsheet for screenshot:

AccessEngineering Excel Spreadsheet for Incompressible Flow in Pipes and Channel

Calculations for pipe systems with minor losses: Example #6, illustrated calculation of straight pipe frictional head loss and frictional pressure drop calculation for a case in which no fittings were present, so there was no need to include minor losses in the calculations. The Figure 7 screenshot shows 0.00 in the cell for the value of **ΣK**. Example #7 illustrates a similar calculation in which fittings (and hence minor losses) are present.

Example #7: What would be the frictional head loss in ft and the frictional pressure drop in psi for 0.9 cfs of water at 50^{o}F flowing through a piping system with 80 ft of 8 inch diameter galvanized iron pipe, containing two medium radius elbows and one fully open gate valve?

Solution: Note that this piping system is the same as that in Example #6, with the addition of the two elbow and one gate valve. The first 6 steps of the Example #6 solution remain the same, leading to a value for the Moody friction factor: $\mathbf{f_m}$ = 0.02085. The next steps then proceed as follows

7. From Table 1, for a medium radius elbow K = 0.8 and for a fully open gate valve K = 0.2. Thus ΣK = 0.8 + 0.8 + 0.2 = 1.8.

8. Substituting specified and calculated values for the parameters in the Darcy Weisbach equation gives: $\mathbf{h_L = [f(L/D) + \Sigma K](V^2/2g)}$

$$= [(0.02085)(80/0.6667) + 1.8][2.58^2/(2*32.2)] = \mathbf{0.4445\ ft = h_L}$$

9. The frictional pressure drop can now be calculated from:

$$\mathbf{\Delta P_f = \rho g h_L} = 1.94*32.174*0.4445 = 27.54 \text{ lb/ft}^2$$

$$= 27.54/144 \text{ psi} = \mathbf{0.1926\ psi = \Delta P_f}$$

As shown by a comparison of the head loss and pressure drop from Examples #6 and #7, the "minor losses" can be a significant part of the frictional head loss/pressure drop.

6. A Spreadsheet for Calculation of Pipe Flow Rate

Determining the flow rate for a specified fluid at known temperature, through a pipe with specified length, diameter and material or surface roughness coefficient, and perhaps minor loss coefficients, is another type of Darcy Weisbach or Fanning equation calculation. Determining flow rate requires an iterative calculation because fluid velocity is needed to calculate **Re** and **Re** is needed to calculate the value of the friction factor, and the friction factor is needed to calculate the fluid velocity and flow rate. An assumed value for flow rate is needed to start the process. An organized approach to this type of calculation with specified fluid temperature, pipe diameter and length, pipe material, allowable head loss, and sum of minor loss coefficients is summarized in the following steps:

1. Obtain values for the density and viscosity of the flowing fluid at its specified temperature.

2. Look up a value for the pipe roughness coefficient, $\boldsymbol{\varepsilon}$, for the specified pipe material.

3. Assume a value for the fluid flow rate.

4. Calculate the fluid velocity using the specified pipe diameter and the assumed fluid flow rate.

5. Calculate the Reynolds number $(\mathbf{Re = DV\rho/\mu})$

6. Calculate either the Fanning friction factor or the Moody friction factor by an iterative calculation using the Churchill equation and Colebrook equation as illustrated in Example #5.

7. Use the Fanning equation or the Darcy-Weisbach equation to calculate the average velocity of the fluid, **V**.

8. Calculate the flow rate of the fluid using $\mathbf{Q = V(\pi D^2/4)}$.

9. If the calculated value of Q is not equal to the assumed value, then the Excel Goal Seek process can be used as described on the spreadsheet to find the assumed value of Q that makes the two equal and is thus the correct answer for the flow rate.

Example #9: Calculate the flow rate of 50°F water due to a head loss of 1.2 ft across 80 ft of 6 inch diameter galvanized iron pipe, with minor loss coefficients totaling 1.8.

Solution: The spreadsheet screenshot in Figure 8 shows the solution to this example. All of the 8 steps shown above are included in the spreadsheet solution.

1. For the Access Engineering pipe flow calculations spreadsheet, which is the source for the screenshot in Figure 8, the first step is completed by selecting a fluid and entering the fluid temperature on a Fluid Properties worksheet. The fluid density and viscosity are then available for use in the other worksheets.

2. The surface roughness for the pipe may be obtained from Table 1 in Chapter1, or from the sources shown for the values in Table 1. The value of ε, needs to be entered into the indicated yellow cell, along with the pipe diameter, pipe length, and allowable head loss, each in its respective yellow cell.

3. A guessed value for the flow rate Q needs to be entered in the indicated cell.

4. The spreadsheet calculates the fluid average velocity using $\mathbf{V = Q/(\pi D^2/4)}$.

5. The spreadsheet calculates the Reynolds number using the known values of **D, V,** ρ and μ.

6. The spreadsheet calculates either the Fanning friction factor or the Moody friction factor as specified by the user with an automated iterative calculation, using the Churchill and Colebrook equations, as illustrated in Examples #4 and #5.

7. The fluid velocity is calculated by the spreadsheet using the Fanning equation or the Darcy Weisbach equation.

8. Now the fluid flow rate is calculated by the spreadsheet using Q = VA.

9. Typically the calculated value of Q will not be equal to the initial assumed value, so the Excel Goal Seek process will need to be carried out as described at the bottom of the screenshot in Figure 8. As shown in Figure 8, the final answer for the flow rate is **0.753 cfs** or **337.8 gpm.**

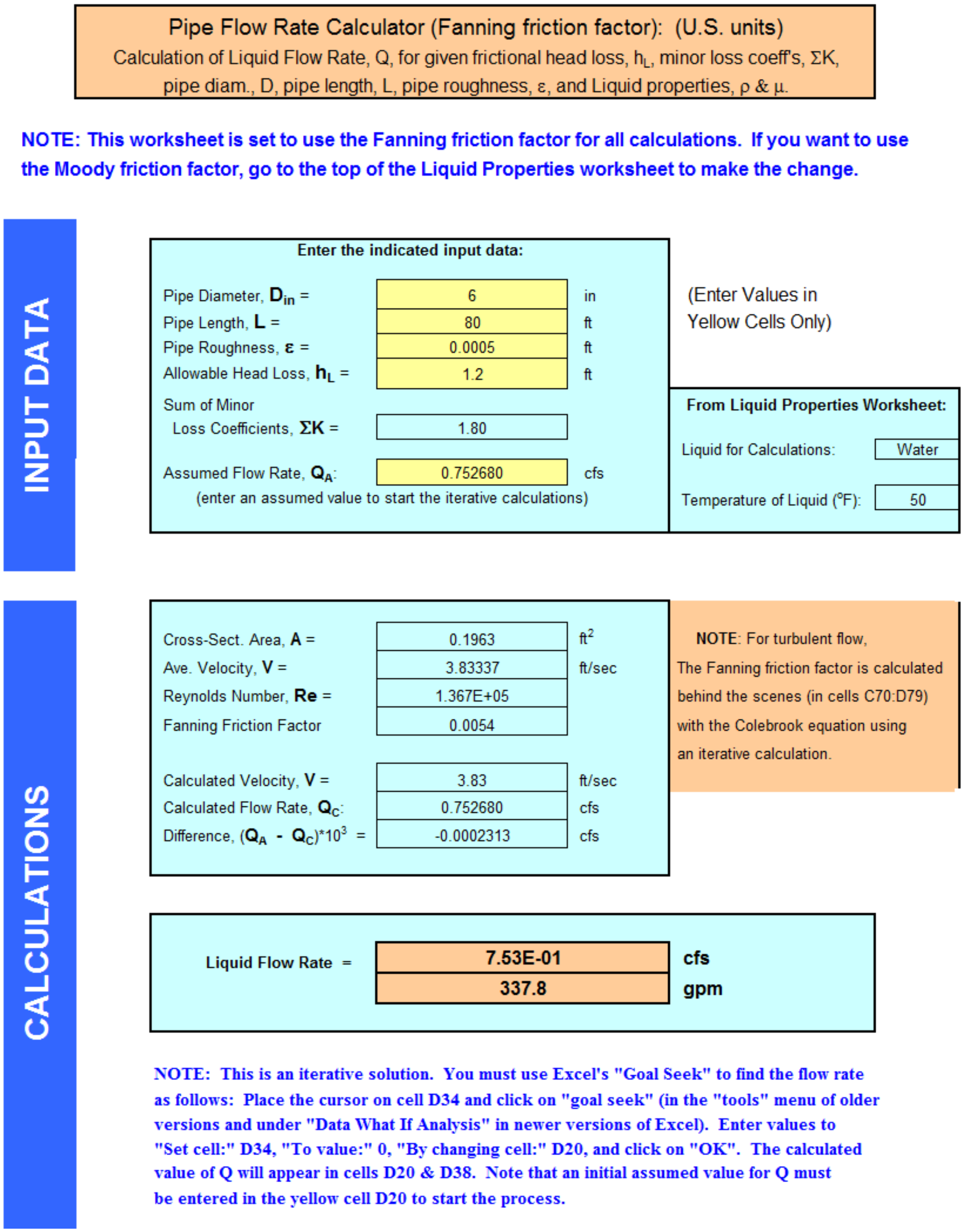

Figure 8. Screenshot of a Flow Rate Calculator Spreadsheet

Source of spreadsheet for screenshot:

[AccessEngineering Excel Spreadsheet for Incompressible Flow in Pipes and Channels]

7. A Spreadsheet for Calculation of Required Pipe Diameter

Spreadsheets also work well for determining the required pipe diameter to carry a given flow rate of a known fluid at specified temperature through pipe of given material and length, given a specified allowable head loss and the sum of minor loss coefficients for the piping system. The Darcy Weisbach equation or the Fanning equation can be used to carry out this type of calculation. An iterative calculation is needed to determine pipe diameter, D, because D is needed to calculate the Reynolds number. Then, within each iteration for D, an iterative calculation of friction factor with the Colebrook equation is needed. The overall process is outlined by the following steps.

1. Find the value of the pipe roughness, $\boldsymbol{\varepsilon}$, for the given pipe material.

2. Get values of the density, $\boldsymbol{\rho}$, and viscosity, $\boldsymbol{\mu}$, for the specified flowing fluid at its operating temperature.

3. Select a pipe diameter, **D**, to use as the starting point for the iterative calculation.

4. Use the assumed pipe diameter, **D**, to calculate the cross-sectional area of flow, the average velocity, and the Reynolds number.

5. Use the value of Reynolds number calculated in step 5, the specified value of $\boldsymbol{\varepsilon}$, and the assumed pipe diameter, to calculate the friction factor ($\mathbf{f_m}$ or $\mathbf{f_f}$) by an iterative process with the Colebrook equation.

6. Use the Fanning equation or the Darcy Weisbach equation to calculate the pipe diameter, using the known values of **L**, $\mathbf{h_L}$ with the calculated values of **V** and friction factor, $\mathbf{f_f}$ or $\mathbf{f_m}$.

7. Typically the calculated value of D will not be equal to the initial assumed value, so the Excel Goal Seek process will need to be carried out as described at the bottom of the screenshot in Figure 9. This process will find the value of D that makes the calculated and assumed values of D equal, which is the required minimum pipe diameter.

Example #10: Find the minimum required pipe diameter to carry 0.80 cfs of water at 50°F through 60 ft of galvanized iron pipe with minor loss coefficients totaling 2.2, if the allowable head loss is 2 ft.

Solution: The spreadsheet screenshot in Figure 9 shows the solution to example #10. All of the 7 steps shown above are included in the spreadsheet solution.

1. From Table 1, the value of **ε** for galvanized iron pipe is 0.0005 ft. This was entered into the appropriate yellow cell, along with the pipe length, allowable head loss, pipe flow rate and an initial assumed pipe diameter.

2. For the Access Engineering pipe flow calculations spreadsheet, which is the source for the screenshot in Figure 8, the second step is completed by selecting a fluid and entering the fluid temperature on a Fluid Properties worksheet. The fluid density and viscosity are then available for use in the other worksheets. Note that **ρ** = 1.940 slugs/ft^3 and **μ** = 2.370 x 10^{-5} lb-sec/ft^2 for water at 50°F.

3. Assume an initial value for pipe diameter: e.g. **D** = 5 inches.

steps 4 - 5: Using the assumed value of D, the spreadsheet calculates the cross-sectional area of flow, the average fluid velocity, the Reynolds number, and either the Moody or Fanning friction factor.

6. The spreadsheet also calculates the pipe diameter using either the Fanning equation or the Darcy-Weisbach equation.

7. Typically the calculated value of D will not be equal to the initial assumed value, so the Excel Goal Seek process will need to be carried out as described at the bottom of the screenshot in Figure 9. This process will find the value of D that makes the calculated and assumed values of D equal, which is the required minimum pipe diameter. In this case the calculated minimum required pipe diameter is **5.40 in.**

Source of spreadsheet for screenshots in Figure 9:

AccessEngineering Excel Spreadsheet for Incompressible Flow in Pipes and Channels

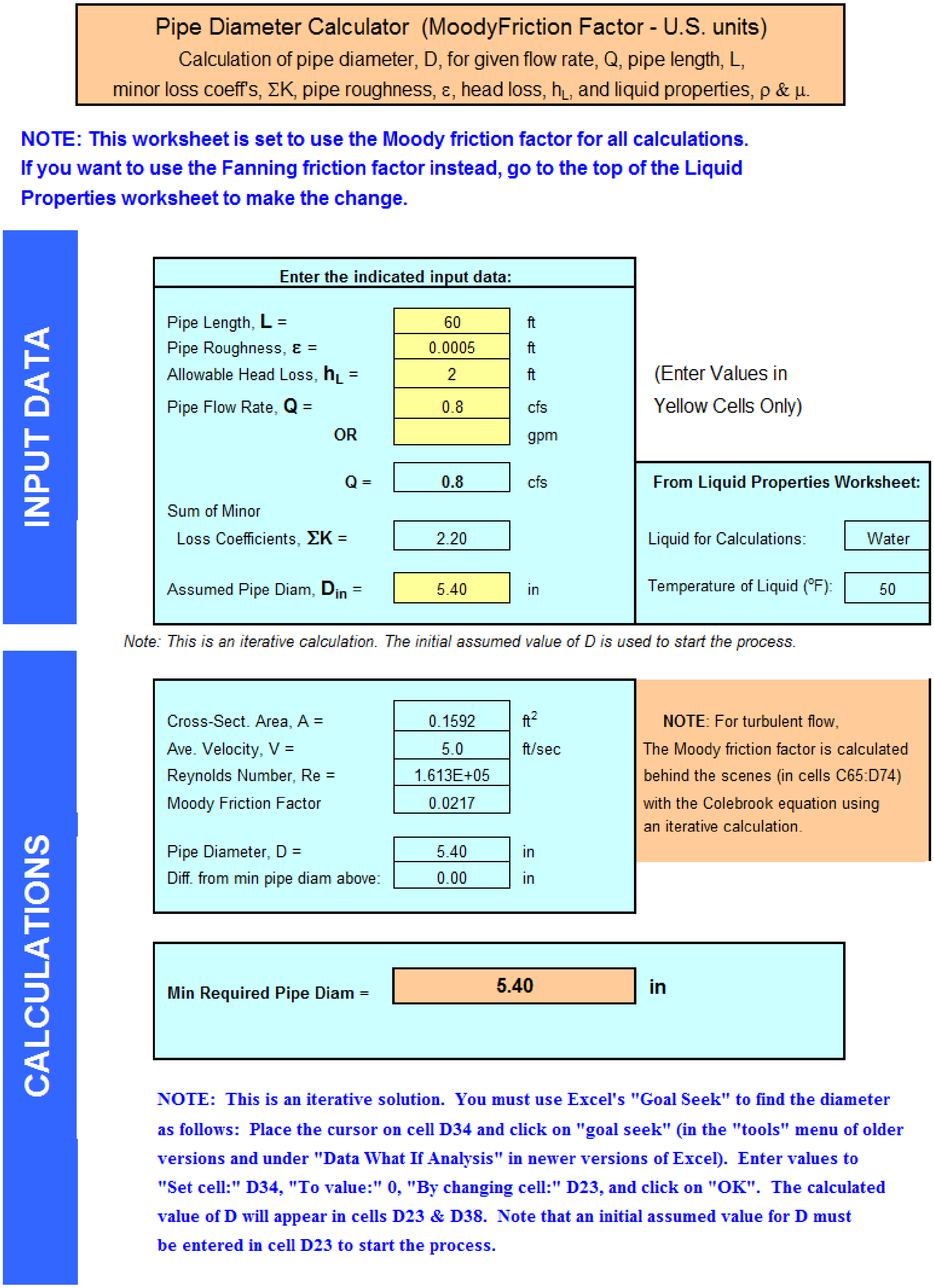

Figure 9. Screenshot of a Pipe Diameter Calculator Spreadsheet

Summary

The Darcy-Weisbach equation or Fanning equation can be used together with the Moody friction factor or the Fanning friction factor to make calculations involving the pipe flow variables, pipe length, **L**; pipe diameter, **D**, pipe roughness, **ε**, fluid flow rate, **Q**; frictional head loss, $\mathbf{h_L}$ or frictional pressure drop, $\mathbf{\Delta P_f}$, average fluid velocity, **V**; sum of minor loss coefficients, **ΣK**, and fluid properties (density and viscosity), as discussed in this tutorial. Spreadsheets are a very convenient method for making these calculations because iterative solutions are required for some of the equations. The three major types of calculations discussed and illustrated with examples and spreadsheet screenshots in this tutorial are i) calculation of frictional head loss or frictional pressure drop, ii) calculation of fluid flow rate, and iii) calculation of minimum required pipe diameter. Each of these can be determined if the other parameters are known.

References

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2. Avallone, E.A., Baumeister III, T. & Sadegh, A., *Marks' Standard Handbook for Mechanical Engineers,* 11th Ed., New York, McGraw-Hill Book Company, 2006. Sec. 3.3.11. Flow in Pipes.

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4. Hicks, T.G., Chopey, N.P., *Handbook of Chemical Engineering Calculations*, 4th Ed., New York, McGraw-Hill Book Company, 2012. Sec. 6.4. Determining the Pressure Loss in Pipes.

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