INTRODUCTION TO OPERATIONS RESEARCH

Eleventh Edition

FREDERICK S. HILLIER
Stanford University

GERALD J. LIEBERMAN
Late of Stanford University

Mc Graw Hill

INTRODUCTION TO OPERATIONS RESEARCH, ELEVENTH EDITION

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ABOUT THE AUTHORS

Frederick S. Hillier was born and raised in Aberdeen, Washington, where he was an award winner in statewide high school contests in essay writing, mathematics, debate, and music. As an undergraduate at Stanford University, he ranked first in his engineering class of over 300 students. He also won the McKinsey Prize for technical writing, won the Outstanding Sophomore Debater award, played in the Stanford Woodwind Quintet and Stanford Symphony Orchestra, and won the Hamilton Award for combining excellence in engineering with notable achievements in the humanities and social sciences. Upon his graduation with a BS degree in industrial engineering, he was awarded three national fellowships (National Science Foundation, Tau Beta Pi, and Danforth) for graduate study at Stanford with specialization in operations research. During his three years of graduate study, he took numerous additional courses in mathematics, statistics, and economics beyond what was required for his MS and PhD degrees while also teaching two courses (including "Introduction to Operations Research"). Upon receiving his PhD degree, he joined the faculty of Stanford University and began work on the 1st edition of this textbook two years later. He subsequently earned tenure at the age of 28 and the rank of full professor at 32. He also received visiting appointments at Cornell University, Carnegie-Mellon University, the Technical University of Denmark, the University of Canterbury (New Zealand), and the University of Cambridge (England). After 35 years on the Stanford faculty, he took early retirement from his faculty responsibilities in order to focus full time on textbook writing, and now is Professor Emeritus of Operations Research at Stanford.

Dr. Hillier's research has extended into a variety of areas, including integer programming, queueing theory and its application, statistical quality control, the application of operations research to the design of production systems, and capital budgeting. He has published widely, and his seminal papers have been selected for republication in books of selected readings at least 10 times. He was the first-prize winner of a research contest on "Capital Budgeting of Interrelated Projects" sponsored by The Institute of Management Sciences (TIMS) and the U.S. Office of Naval Research. He and Dr. Lieberman also received the honorable mention award for the 1995 Lanchester Prize (best English-language publication of any kind in the field of operations research), which was awarded by the Institute for Operations Research and the Management Sciences (INFORMS) for the 6th edition of this book. In addition, he was the recipient of the prestigious 2004 INFORMS Expository Writing Award for the 8th edition of this book.

Dr. Hillier has held many leadership positions with the professional societies in his field. For example, he has served as treasurer of the Operations Research Society of America (ORSA), vice president for meetings of TIMS, co-general chairman of the 1989 TIMS International Meeting in Osaka, Japan, chair of the TIMS Publications Committee, chair of the ORSA Search Committee for Editor of *Operations Research,* chair of the ORSA Resources Planning Committee, chair of the ORSA/TIMS Combined Meetings Committee, and chair of the John von Neumann Theory Prize Selection Committee for INFORMS. He also is a Fellow of INFORMS. In addition, he served for 20 years (until 2013) as the founding series editor for Springer's International Series in Operations Research and Management Science, a particularly prominent book series with nearly

300 published books. In 2018, he was awarded the Kimball Medal (a lifetime achievement award) by INFORMS for his distinguished contributions to the field and to INFORMS.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *The Evaluation of Risky Interrelated Investments* (North-Holland, 1969), *Queueing Tables and Graphs* (Elsevier North-Holland, 1981, co-authored by O. S. Yu, with D. M. Avis, L. D. Fossett, F. D. Lo, and M. I. Reiman), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (6th ed., McGraw-Hill, 2019, co-authored by his son Mark Hillier).

The late **Gerald J. Lieberman** sadly passed away in 1999. He had been Professor Emeritus of Operations Research and Statistics at Stanford University, where he was the founding chair of the Department of Operations Research. He was both an engineer (having received an undergraduate degree in mechanical engineering from Cooper Union) and an operations research statistician (with an AM from Columbia University in mathematical statistics, and a PhD from Stanford University in statistics).

Dr. Lieberman was one of Stanford's most eminent leaders. After chairing the Department of Operations Research, he served as associate dean of the School of Humanities and Sciences, vice provost and dean of research, vice provost and dean of graduate studies, chair of the faculty senate, member of the University Advisory Board, and chair of the Centennial Celebration Committee. He also served as provost or acting provost under three different Stanford presidents.

Throughout these years of university leadership, he also remained active professionally. His research was in the stochastic areas of operations research, often at the interface of applied probability and statistics. He published extensively in the areas of reliability and quality control, and in the modeling of complex systems, including their optimal design, when resources are limited.

Highly respected as a senior statesman of the field of operations research, Dr. Lieberman served in numerous leadership roles, including as the elected president of The Institute of Management Sciences. His professional honors included being elected to the National Academy of Engineering, receiving the Shewhart Medal of the American Society for Quality Control, receiving the Cuthbertson Award for exceptional service to Stanford University, and serving as a Fellow at Stanford's Center for Advanced Study in the Behavioral Sciences. In addition, the Institute for Operations Research and the Management Sciences (INFORMS) awarded him and Dr. Hillier the honorable mention award for the 1995 Lanchester Prize for the 6th edition of this book. In 1996, INFORMS also awarded him the prestigious Kimball Medal for his distinguished contributions to the field and to INFORMS.

In addition to *Introduction to Operations Research* and two companion volumes, *Introduction to Mathematical Programming* (2nd ed., 1995) and *Introduction to Stochastic Models in Operations Research* (1990), his books are *Handbook of Industrial Statistics* (Prentice-Hall, 1955, co-authored by A. H. Bowker), *Tables of the Non-Central t-Distribution* (Stanford University Press, 1957, co-authored by G. J. Resnikoff), *Tables of the Hypergeometric Probability Distribution* (Stanford University Press, 1961, co-authored by D. Owen), *Engineering Statistics*, (2nd ed., Prentice-Hall, 1972, co-authored by A. H. Bowker), and *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets* (McGraw-Hill/Irwin, 2000, co-authored by F. S. Hillier and M. S. Hillier).

ABOUT THE CASE WRITERS

Karl Schmedders is professor of quantitative business administration at the University of Zurich in Switzerland and a visiting associate professor at the Kellogg Graduate School of Management (Northwestern University). His research interests include management science, financial economics, and computational economics and finance. He received his PhD in operations research from Stanford University, where he taught both undergraduate and graduate classes in operations research, including a case studies course in operations research. He received several teaching awards at Stanford, including the university's prestigious Walter J. Gores Teaching Award. After post-doctoral research at the Hoover Institution, a think tank on the Stanford campus, he became assistant professor of managerial economics and decision sciences at the Kellogg School. He was promoted to associate professor in 2001 and received tenure in 2005. In 2008, he joined the University of Zurich, where he currently teaches courses in management science, business analytics, and computational economics and finance. He has published research articles in international academic journals such as *Management Science, Operations Research, Econometrics, The Review of Economic Studies,* and *The Journal of Finance*, among others. He is a co-founder of an EdTech Startup developing a digital learning and grading platform for science education. At Kellogg he received several teaching awards, including the L. G. Lavengood Professor of the Year Award. More recently he has won the best professor award of the Kellogg School's European EMBA program in eight different years, as well as in 2017 for its EMBA program in Hong Kong.

Molly Stephens is a partner in the Los Angeles office of Quinn, Emanuel, Urquhart & Sullivan, LLP. She graduated from Stanford University with a BS degree in industrial engineering and an MS degree in operations research. Ms. Stephens taught public speaking in Stanford's School of Engineering and served as a teaching assistant for a case studies course in operations research. As a teaching assistant, she analyzed operations research problems encountered in the real world and transformed these problems into classroom case studies. Her research was rewarded when she won an undergraduate research grant from Stanford to continue her work and was invited to speak at an INFORMS conference to present her conclusions regarding successful classroom case studies. Following graduation, Ms. Stephens worked at Andersen Consulting as a systems integrator, experiencing real cases from the inside, before resuming her graduate studies to earn a JD degree (with honors) from the University of Texas Law School at Austin. She is a partner in the largest law firm in the United States devoted solely to business litigation, where her practice focuses on complex financial and securities litigation. She also is ranked as a leading securities litigator by Chambers USA (2013 and 2014), which acknowledged "praise for her powerful and impressive securities litigation practice" and noted that she is "phenomenally bright, a critical thinker and great listener."

DEDICATION

To the memory of our parents

and

To the memory of my beloved mentor,
Gerald J. Lieberman, who was one of the true
giants of our field

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PREFACE

When Jerry Lieberman and I started working on the first edition of this book, our goal was to develop a pathbreaking textbook that would help establish the future direction of education in what was then the emerging field of operations research. Following publication, it was unclear how well this particular goal was met, but what did become clear was that the demand for the book was far larger than either of us had anticipated. Neither of us could have imagined that this extensive worldwide demand would continue at such a high level for such an extended period of time.

The enthusiastic response to our first ten editions has been most gratifying. It was a particular pleasure to have the field's leading professional society, the international Institute for Operations Research and the Management Sciences (INFORMS), award the 6th edition honorable mention for the 1995 INFORMS Lanchester Prize (the prize awarded for the year's most outstanding English-language publication of any kind in the field of operations research).

Then, just after the publication of the eighth edition, it was especially gratifying to be the recipient of the prestigious 2004 INFORMS Expository Writing Award for this book, including receiving the following citation:

> Over 37 years, successive editions of this book have introduced more than one-half million students to the field and have attracted many people to enter the field for academic activity and professional practice. Many leaders in the field and many current instructors first learned about the field via an edition of this book. The extensive use of international student editions and translations into 15 other languages has contributed to spreading the field around the world. The book remains preeminent even after 37 years. Although the eighth edition just appeared, the seventh edition had 46 percent of the market for books of its kind, and it ranked second in international sales among all McGraw-Hill publications in engineering.
>
> Two features account for this success. First, the editions have been outstanding from students' points of view due to excellent motivation, clear and intuitive explanations, good examples of professional practice, excellent organization of material, very useful supporting software, and appropriate but not excessive mathematics. Second, the editions have been attractive from instructors' points of view because they repeatedly infuse state-of-the-art material with remarkable lucidity and plain language. For example, a wonderful chapter on metaheuristics was created for the eighth edition.

When we began work on the first edition, Jerry already was a prominent member of the field, a successful textbook writer, and the chairman of a renowned operations research program at Stanford University. I was a very young assistant professor just starting my career. It was a wonderful opportunity for me to work with and to learn from the master. I will be forever indebted to Jerry for giving me this opportunity.

Now, sadly, Jerry is no longer with us. During the progressive illness that led to his death in 1999, I resolved that I would pick up the torch and devote myself to subsequent editions of this book, maintaining a standard that would fully honor Jerry. Therefore, I took early retirement from my faculty responsibilities at Stanford in order to work full time on textbook writing for the foreseeable future. This has enabled me to spend far more than the usual amount of time in preparing each new edition. It also has enabled

me to closely monitor new trends and developments in the field in order to bring this edition completely up to date. This monitoring has led to the addition of a considerable number of important topics to recent editions of the book.

The field continues to evolve fairly rapidly. The most important of the recent developments has been the rise of analytics as a very important complement to operations research. Other important trends also are under way. Therefore, I have made a special effort with this edition to continue bringing this book fully into the 21st century. The many major additions to the new edition are outlined below.

WHAT'S NEW IN THIS EDITION

- Added a New Section 1.3: The Relationship between Analytics and Operations Research.
- Added a New Section 1.5: Some Trends That Should Further Increase the Future Impact of Operations Research.
- Added a New Section 2.2: Gathering and Organizing Relevant Data.
- Added a New Section 2.3: Using Descriptive Analytics to Analyze Big Data.
- Added a New Section 2.4: Using Predictive Analytics to Analyze Big Data.
- Reorganized Section 4.6 (Adapting the Simplex Method to Nonstandard Forms) into Three New Shorter Sections.
- Section 4.10: Added Up-To-Date information on the Factors Affecting the Speed of the Simplex Method (and Its Variants).
- Section 4.11: Added Up-To-Date Information on the Factors Affecting the Relative Performance of the Simplex Method and Interior-Point Algorithms.
- Shortened and Revised Section 12.3: Using Binary Variables to Deal with Fixed Charges.
- Shortened and Revised Section 12.4: A Binary Representation of General Integer Variables.
- Added a New Section 16.7: Multiple Criteria Decision Analysis, Including Goal Programming.
- Added a New Section 17.11: Behavioral Queueing Theory.
- Added a New Section 19.4: Markov Decision Processes in Practice.
- Added a New Section 20.5: Simulation Optimization.
- Added many New Smaller Updates, Including New Application Vignettes and New Selected References.

Reductions to Make Room for All These New Additions:

The first edition of this book was only a little over 600 pages. However, subsequent editions kept growing until it reached 1200 pages with the 7th edition. That is much too large for an introductory textbook, so I have been working ever since to decrease the size of each new edition. I finally got the 10th edition down below 1000 pages again (excluding indices and front matter) and have made a real effort to reduce the size a little further with this new edition. This was a real challenge with all of the new additions outlined above. However, I feel that the reductions listed below have helped to make this a better book by enabling more focus on the important material.

- Dropped Analytic Solver Platform for Education. (This is an excellent software package, but Frontline Systems now is charging students to use it and reviewers expressed little interest in retaining it. This one reduction saved approximately 35 pages.)
- Eliminated an Overabundance of Linear Programming Formulation Examples in Section 3.4. (Dropping three of the six complicated examples saved 10 pages.)

- Shifted Section 6.2 (Economic Interpretation of Duality) to a Supplement on the Website.
- Shifted the General Procedure for Constructing an Initial BF Solution for the Transportation Simplex Method in Section 9.2 to a Supplement on the Website.
- Shifted Most of Section 12.3 (Innovative Uses of Binary Variables in Model Formulation) and Section 12.4 (Some Formulation Examples) to a Supplement on the Website, While Retaining More Elementary Material.
- Deleted a Subsection in Section 17.3 on Outdated Award-Winning Studies That Applied Queueing Theory.
- Deleted 14 Outdated Application Vignettes (While Also Adding 11 New Ones That Are Very Up to Date) and Also Deleted Several Pages of Citations of Outdated Award-Winning OR Applications.

OTHER SPECIAL FEATURES OF THIS BOOK

- **An Emphasis on Real Applications.** The field of operations research is continuing to have a dramatic impact on the success of numerous companies and organizations around the world. Therefore, one of the goals of this book is to tell this story clearly and thereby excite students about the great relevance of the material they are studying. One way this goal is pursued is by including many realistic cases patterned after real applications at the end of chapters and on the book's website. Another way is the inclusion of many application vignettes scattered throughout the book that describe in a few paragraphs how an actual award-winning application of operations research had a powerful impact on a company or organization by using techniques like those studied in that portion of the book. For each application vignette, a problem also is included in the problems section of that chapter that requires the student to read the full article describing the application and then answer some questions. (The only application vignette that lacks this full article is the one in Chapter 1.) The next bullet point describes how students have immediate access to these articles.
- **Links to Many Articles Describing Dramatic OR Applications.** We are excited about a partnership with The Institute for Operations Research and the Management Sciences (INFORMS), our field's preeminent professional society, to provide a link on this book's website to each of the articles that fully describes the application that is summarized in one of the application vignettes. All of these articles appeared in an INFORMS journal called *Interfaces* (now retitled *INFORMS Journal on Applied Analytics* starting in 2019). (Information about INFORMS journals, meetings, job bank, scholarships, awards, and teaching materials is at www.informs.org.) These articles and the corresponding end-of-chapter problems provide instructors with the option of having their students delve into real applications that dramatically demonstrate the relevance of the material being covered in the lectures. It would even be possible to devote significant course time to discussing real applications.
- **A Wealth of Supplementary Chapters and Sections on the Website.** In addition to the nearly 1,000 pages in this book, another several hundred pages of supplementary material also are provided on this book's website (as outlined in the table of contents). This includes eight complete chapters, 12 supplements to chapters in the book, and dozens of additional cases. Most of the supplementary chapters include problems and selected references. Most of the supplements to chapters also have

problems. Today, when students think nothing of accessing material electronically, instructors should feel free to include some of this supplementary material in their courses.

- **Many Additional Examples Are Available.** An especially important learning aid on the book's website is a set of Solved Examples for almost every chapter in the book. We believe that most students will find the examples in the book fully adequate but that others will feel the need to go through additional examples. These solved examples on the website will provide the latter category of students the needed help, but without interrupting the flow of the material in the book on those many occasions when most students don't need to see an additional example. Many students also might find these additional examples helpful when preparing for an examination. We recommend to instructors that they point out this important learning aid to their students.
- **Great Flexibility for What to Emphasize.** We have found that there is great variability in what instructors want to emphasize in an introductory OR survey course. They might want to emphasize the mathematics and algorithms of operations research. Others will emphasize model formulation with little concern for the details of the algorithms needed to solve these models. Others want an even more applied course, with emphasis on applications and the role of OR in managerial decision making. Some instructors will focus on the deterministic models of OR, while others will emphasize stochastic models. There also are great differences in the kind of software (if any) that instructors want their students to use. All of this helps to explain why the book is a relatively large one. We believe that we have provided enough material to meet the needs of all of these kinds of instructors. Furthermore, the book is organized in such a way that it is relatively easy to pick and choose the desired material without loss of continuity. It even is possible to provide great flexibility on the kind of software (if any) that instructors want their students to use, as described below in the section on software options.
- **A Customizable Version of the Text Also is Available.** Because the text provides great flexibility for what to emphasize, an instructor can easily pick and choose just certain portions of the book to cover. Rather than covering most of the pages in the book, perhaps you wish to use only a much smaller portion of the text. Fortunately, McGraw-Hill provides an option for using a considerably smaller and less expensive version of the book that is customized to meet your needs. With McGraw-Hill Create™, you can include only the chapters you want to cover. You also can easily rearrange chapters, combine material from other content sources, and quickly upload content you have written, like your course syllabus or teaching notes. If desired, you can use Create to search for useful supplementary material in various other leading McGraw-Hill textbooks. For example, if you wish to emphasize spreadsheet modeling and applications, we would recommend including some chapters from the Hillier-Hillier textbook, *Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheets*. (That textbook includes a complete coverage of the Analytic Solver Platform for Education software package that has been dropped in this edition.) Arrange your book to fit your teaching style. Create even allows you to personalize your book's appearance by selecting the cover and adding your name, school, and course information. Order a Create book and you'll receive a complimentary print review copy in 3–5 business days or a complimentary electronic review copy (eComp) via e-mail in minutes. You can go to www.mcgrawhillcreate.com and register to experience how McGraw-Hill Create empowers you to teach your students your way.

A WEALTH OF SOFTWARE OPTIONS

A wealth of software options is provided on the book's website www.mhhe.com/hillier11e as outlined below:

- Excel spreadsheets: state-of-the-art spreadsheet formulations in Excel files for all relevant examples throughout the book. The standard Excel Solver can solve all of these examples.
- A number of Excel templates for solving basic models.
- Student versions of LINDO (a traditional optimizer) and LINGO (a popular algebraic modeling language), along with formulations and solutions for all relevant examples throughout the book.
- Student versions of MPL (a leading algebraic modeling language) along with an MPL Tutorial and MPL formulations and solutions for all relevant examples throughout the book.
- Student versions of several elite MPL solvers for linear programming, integer programming, convex programming, global optimization, etc.
- Queueing Simulator (for the simulation of queueing systems).
- OR Tutor for illustrating various algorithms in action.
- Interactive Operations Research (IOR) Tutorial for efficiently learning and executing algorithms interactively, implemented in Java 2 in order to be platform independent.

Numerous students have found OR Tutor and IOR Tutorial very helpful for learning various OR algorithms. When moving to the next stage of solving OR models automatically, surveys have found instructors almost equally split in preferring one of the following options for their students' use: (1) Excel spreadsheets, including Excel's Solver, (2) convenient traditional software (LINDO and LINGO), and (3) other state-of-the-art OR software (MPL and its elite solvers). For this edition, therefore, I have retained the philosophy of the last few editions of providing enough introduction in the book to enable the basic use of any of the three options without distracting those using another, while also providing ample supporting material for each option on the book's website.

There are only two software packages that accompanied the 10th edition that are not continued with this new edition. One is the Analytic Solver Platform for Education (ASPE) previously discussed in Sec. 3.5 and several subsequent places. The other is the TreePlan software for decision trees that was described in a supplement to Chapter 16. Our policy is that students must be able to use all the software provided with the book for their course work without any additional charge, but the owners of these two packages now are charging students for their use.

Additional Online Resources

- A *glossary* for every book chapter.
- *Data files* for various cases to enable students to focus on analysis rather than inputting large data sets.
- A *test bank* featuring moderately difficult questions that require students to show their work is being provided to instructors. Many of the questions in this test bank have previously been used successfully as test questions by the authors.
- A *solutions manual* and *image files* for instructors.

THE USE OF THE BOOK

The overall thrust of all the revision efforts has been to build upon the strengths of previous editions to more fully meet the needs of today's students. These revisions make the book even more suitable for use in a modern course that reflects contemporary practice in the field. The use of software is integral to the practice of operations research, so the wealth of software options accompanying the book provides great flexibility to the instructor in choosing the preferred types of software for student use. All the educational resources accompanying the book further enhance the learning experience. Therefore, the book and its website should fit a course where the instructor wants the students to have a single self-contained textbook that complements and supports what happens in the classroom.

The McGraw-Hill editorial team and I think that the net effect of the revision has been to make this edition even more of a "student's book"—clear, interesting, and well-organized with lots of helpful examples and illustrations, good motivation and perspective, easy-to-find important material, and enjoyable homework, without too much notation, terminology, and dense mathematics. We believe and trust that the numerous instructors who have used previous editions will agree that this is the best edition yet.

The prerequisites for a course using this book can be relatively modest. As with previous editions, the mathematics has been kept at a relatively elementary level. Most of Chaps. 1 to 15 (introduction, linear programming, and mathematical programming) require no mathematics beyond high school algebra. Calculus is used only in Chap. 13 (Nonlinear Programming) and in one example in Chap. 11 (Dynamic Programming). Matrix notation is used in Chap. 5 (The Theory of the Simplex Method), Chap. 6 (Duality Theory), Chap. 7 (Linear Programming under Uncertainty), Sec. 8.4 (An Interior-Point Algorithm), and Chap. 13, but the only background needed for this is presented in Appendix 4. For Chaps. 16 to 20 (probabilistic models), a previous introduction to probability theory is assumed, and calculus is used in a few places. In general terms, the mathematical maturity that a student achieves through taking an elementary calculus course is useful throughout Chaps. 16 to 20 and for the more advanced material in the preceding chapters.

The content of the book is aimed largely at the upper-division undergraduate level (including well-prepared sophomores) and at first-year (master's level) graduate students. Because of the book's great flexibility, there are many ways to package the material into a course. Chapters 1 and 2 give an introduction to the subject of operations research. Chapters 3 to 15 (on linear programming and mathematical programming) may essentially be covered independently of Chaps. 16 to 20 (on probabilistic models), and vice-versa. Furthermore, the individual chapters among Chaps. 3 to 15 are almost independent, except that they all use basic material presented in Chap. 3 and perhaps in Chap. 4. Parts of Chapters 5-8 are a little more challenging mathematically than the prior chapters. Chapters 6 and 7 and Sec. 8.2 draw upon Chap. 5. Sections 8.1 and 8.2 use parts of Chaps. 6 and 7. Section 10.6 assumes an acquaintance with the problem formulations in Secs. 9.1 and 9.3, while prior exposure to Secs. 8.3 and 9.2 is helpful (but not essential) in Sec. 10.7. Within Chaps. 16 to 20, there is considerable flexibility of coverage, although some integration of the material is available.

An elementary survey course covering linear programming, mathematical programming, and some probabilistic models can be presented in a quarter (40 hours) or semester by selectively drawing from material throughout the book. For example, a good survey of the field can be obtained from Chaps. 1, 2, 3, 4, 16, 17, 18, and 20, along

with parts of Chaps. 10 to 14. A more extensive elementary survey course can be completed in two quarters (60 to 80 hours) by excluding just a few chapters, for example, Chaps. 8, 15, and 19. Chapters 1 to 9 (and perhaps part of Chap. 10) form an excellent basis for a (one-quarter) course in linear programming. The material in Chaps. 10 to 15 covers topics for another (one-quarter) course in other deterministic models. Finally, the material in Chaps. 16 to 20 covers the probabilistic (stochastic) models of operations research suitable for presentation in a (one-quarter) course. In fact, these latter three courses (the material in the entire text) can be viewed as a basic one-year sequence in the techniques of operations research, forming the core of a master's degree program.

The book's website will provide any updates about the book, including an errata. To access this site, visit www.mhhe.com/hillier11e.

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I am particularly grateful to three friends who provided expert advice on specific topics for this edition. I have known all of them well since they were students (and eventually PhD graduates) at Stanford a few decades ago, and all three have gone on to illustrious careers in the field. One is Irv Lustig, who currently is an Optimization principal with Princeton Consultants. Irv is well known as being on the leading edge of current developments at the interface between theory and practice, including in the area of analytics. A second is Vijay Mehrotra, a faculty member at the University of San Francisco who is a regular columnist for the *Analytics* magazine. The third is Edward Rothberg, who is the CEO and a leading computational scientist for GUROBI, a particularly prominent OR software company. Irv and Vijay guided me through the process of developing the four new up-to-date sections on analytics in the first two chapters. Ed identified the current state of the art for me regarding the factors affecting the speed of the simplex method (and its variants), as well as the factors affecting the relative performance of the simplex method and interior-point algorithms. This provided authoritative updates for Sections 4.10 and 4.11.

I also am very fortunate to have a strong team who contributed to recent editions in ways that supported the current edition as well. Our case writers, Karl Schmedders and Molly Stephens (both graduates of our department), wrote 24 elaborate cases for the 7th edition, and all of these cases continue to accompany this new edition. One of our department's former PhD students, Michael O'Sullivan, developed OR Tutor for the 7th edition (and continued here), based on part of the software that my son Mark Hillier had developed for the 5th and 6th editions. Mark (who was born the same year as the first edition, earned his PhD at Stanford, and now is a tenured Associate Professor of

Quantitative Methods at the University of Washington) provided both the spreadsheets and the Excel files (including many Excel templates) once again for this edition, as well as the Queueing Simulator. He also contributed greatly to Chap. 21 on the book's website. In addition, he updated both the 10th edition and the current 11th edition versions of the solutions manual. Earlier editions of this solutions manual were prepared in an exemplary manner by a long sequence of PhD students from our department, including Che-Lin Su for the 8th edition and Pelin Canbolat for the 9th edition. Che-Lin and Pelin did outstanding work that nicely paved the way for Mark's work on the solutions manual. Last, but definitely not least, my dear wife, Ann Hillier (another Stanford graduate with a minor in operations research), provided me with important help on a regular basis. All the individuals named above were vital members of the team.

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I also am excited about the partnership with INFORMS that began with the 9th edition. Students can benefit greatly by reading about top-quality applications of operations research. This preeminent professional OR society is enabling this by providing a link to the articles in *Interfaces* (now called *INFORMS Journal on Applied Analytics*) that describe the applications of OR that are summarized in the application vignettes provided in the book.

It was a real pleasure working with McGraw-Hill's thoroughly professional editorial and production staff, including Theresa Collins (the Product Developer during most of the development of this edition), and Jason Stauter (Content Project Manager).

Just as so many individuals made important contributions to this edition, I would like to invite each of you to start contributing to the next edition by using my email address below to send me your comments, suggestions, and errata to help me improve the book in the future. In giving my email address, let me also assure instructors that I will continue to follow the policy of not providing solutions to problems and cases in the book to anybody (including your students) who contacts me.

Enjoy the book.

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